Using ICTs to Facilitate Multilingual Mathematics Teaching and Learning

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Abstract

Many mathematics teachers and learners are living in a world where Information and Communication Technologies (ICTs), including computers and mobile phones, are common – they are digital natives. Many devices of the information and communication technologies are also connected to the Internet, meaning that an unprecedented audience and unparalleled knowledge can both be reached: the world is connected far beyond language, country or social boundaries. ICTs offer access to numerous online knowledge sources, several of which are multilingual. ICTs also connect a large number of learners to text discussions of mathematics. The community exposure or peer-to-peer nature of this communication allows it to be in a language that is highly relevant for these learners and enables them to use their own language within a shared repertoire. Finally, ICTs support exploration of mathematical objects: they commonly transcend languages through the expressivity and interactivity of representations they offer to view and manipulate mathematical ideas. This chapter surveys research that studies such innovations in the ICTs and mathematics education literature. It pinpoints the gap related to the scarcity of literature on the possibilities of using ICTs to facilitate multilingual mathematics teaching and learning. To this end, the first section reviews literature about ICTs for mathematics learning, identifying a few opportunities where the language diversity is considered. It then describes how multilingualism can affect tools of the ICTs for mathematics learning. Finally it describes a few tool types, which may support the teaching and learning mathematics in multilingual environments.

1. Consideration for the Language Diversity in the Literature of ICTs for Mathematics Teaching and Learning

The field of ICTs in mathematics learning has undergone multiple investigations and is still an important area for ongoing research. Mathematics education conferences, such as the Congress of European Research in Mathematics Education (CERME), often include *Working Groups* on the usage of ICTs in mathematics teaching and learning. Research published in this area of scholarship includes the presentation of new learning tools, design methods, training or support concepts, the investigation of methods to enhance the quality of the usage of the tools, and results on the impact of the use of computers in mathematics teaching. In this section, we review studies about mathematics education with technology that inform our main aim for this chapter.

Borba, Clarkson and Gadanidis (2012) provide evidence that new practices are enabled by the introduction of information technologies in teaching. In particular, they sketch teaching practices, which allow a much richer communication, either peer to peer or in the form of performances. From observations such as these, one sees that the introduction of ICTs into mathematics teaching brings different ways to express and perceive the mathematical activities, concepts, and phenomena. One can thus expect the use of ICTs to offer multiple new opportunities for learners to employ the different languages of their environments. However, the literature about ICTs in mathematics learning does not contain, yet, many reports that show support for the usage in multilingual environments.

One study that contributes to this support was the ICMI Study 17 (Hoyles & Lagrange, 2010). This focused on the usage of technologies in mathematics education and provided a broad 'state of the art' survey of the field, which we shall employ in this chapter. Drijvers, Kieran and Mariotti (2010) as part of the study surveyed the theoretical frameworks that are applicable to learning with digital tools and summarize their contribution noting that "some aspects remain underexposed, such as the role of language in instrumental genesis" (p. 121). Indeed a simple text search through the study book shows that the word *language* is used very often to denote other meanings than that used among persons every day, such as a symbolic or programming language (in about 40% of the occurrences). Even the chapters on inclusion and equity barely addressed the inclusion issues that are typical in western classrooms where the language barriers of children of immigrants' families meet challenges by not mastering the language of teaching. Hence even in this definitive study of ICT and mathematics education, the issues of learning in a multi lingual context rarely rated a mention. The remainder of this section will highlight some aspects of research addressing the transformed communication between multiple persons induced by tools of ICTs, followed by the research towards the learning affordances that ICTs are known to offer.

1.1 ICTs-Mediated Human Communication for Maths Learning

Endrizzi (2012) published a broad literature review of the French and English research literatures concerning the usage of computer-based-tools in higher education. The review underlined the scarce literature reporting on usage of computers for learning. The report also underlined the emergence of newer forms of didactical organizations, notably the *flipped classroom*, where the plain dissemination-like courses are relayed to online videos, and all in-presence meetings in a classroom or tutorial room are used for opportunities to exchange, ask, and collaborate. These new ways for didactical organization employ technology as a central means of communication, which in this context is at least as important as the textbook is in most mathematics classes in industrialized countries. Although there is little in the literature that delves into the affordances or disadvantages of such an approach, one feature is directly relevant for this chapter: the more individualized nature of the communication is likely to impact on the usage of different languages. It allows, for example, groups of speakers of a different language than the language of teaching to communicate in their own language. This context is similar to the study of Setati, Molefe and Langa (2008) where students were assigned to groups sharing a home language and were shown to transparently leverage both the language of teaching (English) and their home language.

The report of Endrizzi (2012) also provides hints on how recommendations of the usage of online learning resources were exchanged and received. It noted that, while multiple learning resources were available for many of the topics to be learned, the choice to use them was rarely reflected and was stimulated by two factors: firstly the necessity to work with a group of peers, and secondly to take into account the recommendations of the teachers. This is important for teachers in a multilingual environment because it shows the importance for them to maintain a collection of quality resources, which are likely to help each of the learners taking their differences in account be them language, affinities, or expectations. The availability of such a collection of recommendation should be compared to the current practice of students that are more comfortable in a different language than that of teaching and gather their own collections by simply crawling the web.

Beaty and Geiger (2010), in a contribution to the ICMI Study 17 underlined the novel collaboration possibilities enabled by digital technologies, and supported by multiple social learning theories. The new social dynamics of these collaboration methods is likely to exploit the language diversity in a different way than a classroom where a single language is often the best choice to ensure a consistent presentation. Beaty and Geiger, citing Sfard, explained how “learning mathematics is an initiation into a certain well-defined discourse“ which technology tools can carry. They described scenarios of computer supported collaborative learning (CSCL), but did not describe the potential effect in multilingual classrooms, a sad omission.

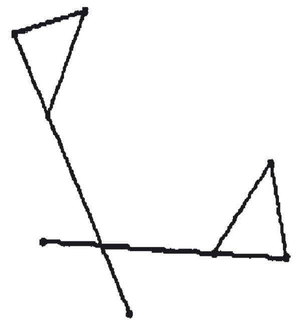
1.2 With and Beyond the Language in Learning Tools

Learning tools within ICTs employs a language, including words and mathematical formulæ, to display the mathematical concepts they manipulate. However, that language is generally not as rich as that of a teacher or of classroom peers, and it often includes symbolic or graphical representations. This is noted by Hardman (2005) who stated that, “In the computer laboratory … there is less reliance on language as a tool to explain mathematical content, and more reliance on language as a tool to regulate behaviour” (Hardman, 2005, p. 10).

Banyard, Underwood and Twiner (2006, p. 482) studied usage of computers “at an inner city primary school … in central London” where 57% of the learners had “English as an additional language” and 40 registered languages were spoken in the school. They noted “that the ICT rich learning environment removed the language hurdle that many children experience in their school work.” In contrast Kozma (2005, p. 15) indicated that there is a body of consistent evidence that indigenous minority language speaking learners “all experience growth in their sense of self-esteem and autonomy in their learning when given access to computers in the context of student-centred pedagogy.”

Drijvers et al. (2010), having introduced several theoretical frameworks to describe learning processes with computers, citing Noss and Hoyles, gave the example of two girls Cleo and Musha, who employed a dynamic geometry system to solve the task of finding where the symmetry axis of two flags was when they were a reflection of each other (see Figure 1). The girls solved the exercise with this sentence: “The mirror line is what you see on the screen if you drag points and their reflection together” (p.101). The learning tool gave them the opportunity to explore the geometry freely and formulate in their own words their understanding of a solution.

Olive, Makar, Hoyos, Kee Kor, Kosheleva and Sträßer (2010), also in the ICMI Study 17, studied the representations of knowledge in the technological tools and discussed how the representations that are *inside* the tools map related to the conceptual representations of the learners. As an example, they showed that dynamic geometry software allows learners to attempt (almost) *all* triangles by simply dragging. This gives the learners an opportunity to illustrate multiple cases without having to use, and maybe be impeded by, the language, which would otherwise be necessary to discuss the various triangles by naming them so as to show them.



The studies cited here are a few examples of those that have studied the impact of ICTs in mathematics learning. These studies show that the diversity of the learners' interactions with the computers, compared to classroom interactions, brings new opportunities to build understanding. But as indicated earlier, few of them highlighted the opportunities of learning within a multilingual environment. We have not found any that have systematically measured the impact of ICTs on learning in multilingual environments. Having noted the scarcity of such literature, we now explore how multilingualism is considered in the ICTs literature in general before returning to the opportunity to support mathematics learning in a multilingual environment in particular in the last section.

2. Multilingualism in the ICT Literature

The main focus of multilingualism in the ICT literature is on the ability to exercise multiple languages for any given software. Most of the software economy is at a global scale and it is common to develop software packages so that interaction with them can occur in several languages. The process of refactoring and translating software to a new language is called *localization*.

The dimensions of localization have been studied by multinational corporations. Among the best-known approaches to what needs to be localized are those developed by Hofstaede (1991). After interviewing IBM employees across the world, he developed a set of dimensions, which change according to the culture. Software designers used these extensively in the localization process. The dimensions Hofstaede developed qualify perceptions of social relationships and are summarized on his website[1]. From these relationships, recommended user-interface adjustments can be formulated. A panorama of issues that may be involved in localization (including the dimensions of Hofstaede) is provided in a report of Marcus (2008):

- Typographical aspects (e.g. the rendering of the decimal or thousands separator, including differences in mathematical notations).
- Verbal aspects (the use of different words).
- Symbolic aspects (e.g. the immediate recognisability of a power outlet sign), which can go as far as employing the right colour to stimulate a particular reaction.
- Placement aspects (what is easily found where, on a screen).

While the report of Marcus is not entirely backed by a systematic ethnographical analysis for each differences' dimensions, it echoes a rich experience of practical software localization.

1 http://geert-hofstede.com/dimensions.html

Several refinements of the dimensions of Hofstaede are being attempted, including an exploration of how the *adaptability* of learning software is perceived (Stewart, 2012), and to explore the varying degrees of readiness to disclose personal information in different cultures (Blanchard, 2012). The Hofstaede dimensions provide a basic estimate, but they are not sufficient to create fully relevant software. For this localization teams still need to evaluate and revise software language-by-language (that is communicative language not software language), country-by-country, and culture-by-culture.

Software localization generally fits in the design of a model-view-controller architecture, which is one of the main software design patterns. For the most elementary localization, code for *views* is written, where each *message* (any piece of text) is transformed through its translation using a dictionary of localization messages. For this case, the translators' work is to write the translations of each message. These dictionaries include all the textual messages and also include the choices of colours, the patterns for numbers, or the choice of symbols to denote particular functions. However the dictionary approach is insufficient to tackle more complex cases; for example the display, input, and validation, of a postal address will need different algorithms if done in Canada where the zip code is a sequence of letters and numbers, compared to France where the zip code is made by five digits, or in Switzerland where the zip code is made by four digits.

Localization is often not fully achieved. Kleiner (2012) indicated that while the ability to speak a local language is a requirement for any company wanting to reach more than a local customer base, it seems a challenge not yet faced commonly to translate e-commerce websites across European countries: only 2% of the e-commerce websites are speaking more than four languages (while Europe has 27 main languages). The same can be said of education. One example is that of Dalvit, Thinyane, Muyingi & Terzoli (2007, p. 13) who noted that "many projects involving the implementation of ICTs in rural areas in Africa have failed because of the language barrier posed by the use of English." These authors experimented "with the use of both English and isiXhosa, the local African language". This included "the development and use of teaching material in isiXhosa, to be used alongside the existing material in English." This initiative gave rise to a model of ICT deployment in rural areas, which appears to represent best practice.[2]

From the above, we see that the adaptability of software to different language is a documented process but that it may be difficult to reach in full generality. The next section explores the localization achievements for various software types relevant to mathematics education.

2.1 Mathematics Learning Tools with some Multilingualism

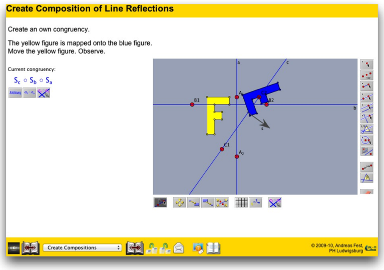
Having described some aspects from recent research on localizing ICTs, we now describe various mathematics learning tools that support some contexts of multilingualism. Mathematics learning software that has attempted to speak multiple languages include:

- Dynamic geometry software and other "simple" manipulation software: for most of these, the names of operations are translated with simple dictionaries. Most also offer a small set of

2 More about this initiative can be read at http://siyakhulall.org .

notational adjustments, which depend on the (mathematical) culture and purposes of the user.

- More specialized tools such as MoveIt-M (Fest, 2011) support a similar localization but specific refinement of language is needed for particular languages. For MoveIt-M, for example, the code which produces a verbal description of a transformation given by its fixed points set, needs adaptation for each new language to produce the appropriate word sequence. The different representations of the glide-reflection are shown in the Figure 2.



Glide reflection in direction of s

$S_c \circ S_b \circ S_a$

- Pocket calculators have started to display localized behaviour from the simple distinction between the "," and "." as decimal separator, through to the name of particular functions (e.g. *gcd* in English and *ggT* in German, *tg* in French and *tan* in German and English).
- The interactive exercisers of the WebALT project have been designed to achieve multilingualism at scale. They are based on an abstract representation of the languages and of the interactive exercises. Caprotti and Seppälä (2006, p. 3) indicated that:

> [these] exercises are produced by the authors using WebALT software that allows creating a language-independent representation of the kind of sentences used in the statements of typical mathematical problems… By allowing the student to view the exercises in a preferred language, multilingual tests in mathematics overcome language barriers in bilingual communities or in communities where there are large minorities speaking a language not supported at schools.

- The ActiveMath web-based environment is a learning environment (Melis, Goguadze, Libbrecht & Ullrich, 2009) providing texts, tools, and interactive exercises for learners. ActiveMath's content is made of texts in multiple languages with mathematical formulæ in an abstract language. It adapts the display of mathematical notations to each language, whether they come from a computation engine, the user input, or the content fragments.

These learning tools illustrate how far reaching learning tools have been able to go so far concerning multilingualism, and how demanding a strong multilingualism could become. In Section 3 other ICTs tools will be described for their potential contribution to the multilingual learners. However before considering more ICT tools another challenge needs to be addressed: the multilingual abilities of search tools, which are an important family of content management tools.

2.2 Search Engines: Beyond a Dictionary of Translations

Although translating the user-interface messages from one language to another is an important first step, it is not sufficient to fully localize software. One of the further challenges is the development of search tools that work well in multiple languages. There seems to be little research into this issue: "Multilingual search, although sophisticated algorithmically, is not yet interesting from an interface perspective, but this may change with time" (Hearst 2009, sec. 12.5). As a rare work in this direction, the book of Peters, Braschler and Clough (2011) sketch out how current cross-lingual search engines can be built and evaluated. In most cases, such search engines are based on the availability of parallel corpora of documents in all considered languages, which are often created using automatic translators.

From the perspective of this chapter it is a pity that Peters et al make few references to students. They do cite Wu et al. indicating that "their findings highlighted the wide use of multilingual resources by Chinese students" (p. 192) and Clough and Eleta showing "the potential benefit of multilingual information access for international students studying abroad", but clearly these are very general references to education, although it is clear that search tools are one class of ICT application that are crucial for multilingual students. Peters et al cited no studies involving multilingual search tools that supported the learning or teaching of mathematics. We suspect the inability of current automatic translation tools to produce satisfactory learning texts in mathematics to be one of the reasons of this.[3]

The role of a search tool is to present a sequence of relevant matches based on a few words of queries. Multilingual challenges that need to be considered in a search tool targeting mathematics in particular include:

- Different algorithms are used for different languages (for example, *sitting* would be equivalent to the word *sit* in an English search engine, but is a word in French with a separate meaning, close to *demonstration*, and hence is treated differently compared to how it is in English).
- Some words have a different semantic field, and thus a different importance, in different languages. For example the term *direction* may have a didactical meaning in English, but has none in French.
- Concepts made of multiple words are often used in mathematics. A good search engine should spot these concepts so that variants are found (e.g. *natural numbers* and the mathematical symbol , or *the right angled triangle* and *this triangle is right angled*).

Differences in the ways search engines behave between languages are evident in widespread web search engines such as Google.com or Bing.com. For example, with Google selected for French, inputting *implication* (which has the same meaning in English) will automatically match the verb *impliquer* (imply) and will also suggest related searches (in logic, in law...). However, inputting the word *hlephula* (implication in Siswati) with a browser configured to prefer Siswati will only show exact matches. Thus the search engines have a different ease of use depending on the language. To resolve such differences, the language technology research community is progressively assembling *language resources* so as to encode the knowledge that supports such features in multiple languages, but there is still a long way to go.

This challenge of searching for mathematical information in multiple languages is well illustrated by the rich online service Wolfram|Alpha.[4] For it to speak another language, it should, for example, also index datasets present in other cultures (e.g. nutrition information in some cultures of that language, which is far from transformable faithfully to the US food facts). As well it should present the formula codified appropriately in the target language, and

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3 Although no formal research has been made on the validity of automatic translators for mathematical texts, we note that challenges such as the translation of the *theorem of Thales* is done by all automatic translators we found into French as the *théorème de Thalès*, whereas these two theorems do not state the same fact (the first states that points on a circle span a right angle to the ends of the diameter, the second is the intercepting lines theorem, stating proportionality of measures. While the knowledge to perform such translation may emerge, we have not observed projects that aim at the completion of such a task.

4 The web site www.wolframalpha.com is a combination of a search engine and a computer algebra interpreter. Each query is translated into a parametrized Mathematica programme which is displayed to the user.

it should also introduce parsing rules for that language. This might weaken the fine tolerance currently available. For example, the input *triangle rectangle* is understood in English as two figure names and thus displays information comparing these two figures side by side (as in the query, *triangle vs rectangle*). However if French was fully supported, it would be understood as the query for *right angled triangle*.

The above brief discussion of different software aspects that are used to support multiple languages has mapped out some of the spectrum of software diversity and the inherent difficulties still present that learners and teachers in multilingual environments are likely to meet. But the scene is not completely devoid of good news. We now turn to the features of currently available software and describe how they can support at least to some degree the learning process in multilingual contexts.

3. Supporting Mathematics Teaching and Learning in Multilingual Environments

In this section, we describe a few available methods where ICTs can help mathematics learners and teachers in a multilingual environment. This description is complementary to the previous sections in that it focuses on the role to support the learners or teachers instead of the computer-specific or learning-specific aspects. We highlight the present immaturity of common knowledge about the need of multilingual learning classrooms: for example only incomplete support is given for the diverging mathematical traditions supported within each tool. In most cases, learners need to be self informed of such differences, and adapt themselves to the language of the ICTs tool.

3.1 Language Switch

For multilingual learners, the ability of a software application to speak multiple languages offers them a flexibility that may support them. For example a student with a math background in one language, but now in a classroom where the teaching language is another, would be helped by performing the exercise first in his own language, to leverage previous knowledge, then moving to the language of the classroom to perform as well as his peers.[5] This requires a language switch to be accessible. Such a switch constitutes a way for learners to employ any of their languages as a *transparent resource*, similarly to the bilingual tasks described in Setati et al. (2008). She describes how learners were given a statement both in their home language and in the language of teaching and they solved the problem in part by discussing the solution process in a mix of the language of teaching and in their home language.

In rare cases in the ICT environment, the users are able to activate a change of the language with a simple action. This keeps all possible states, translating any feedback and any user input that would support the student. Hence parts of an exercise where background knowledge is useful could be entered in the student's own language, and then the process continued in the different language of teaching. Thus far, this is only possible within the ActiveMath environment (Libbrecht, 2010a) and some dynamic geometry software applications (see below). Very little is known of the potential advantages of such language switches for multilingual learners in the ICT environment, although this is a potentially fruitful area of research. These actions are related to the practice of code-switching, which another

5 See a number of other chapters in this volume that address this issue extensively.

chapter in this volume addresses in detail (Goosen et al., 2014). We will therefore not explore the notion of switching any further, but move directly to the context of computer use.

One should note that the existence of such a language switch imposes a parallelism between the languages, which is sometimes difficult to achieve. As expressed in Melis et al. (2009), such concepts as the *instant slope* in the English language are not fully translatable to the French language: the only correct translation would be the *pente de la tangente* (the *slope of the tangent to the curve*) which carries a completely different set of prerequisites and thus would be connected to completely other concepts in an explanation, demonstration, or a navigation through knowledge.

3.2 Multilingualism in Dynamic Geometry Systems

Dynamic geometry software and other interactive learning tools are often able to speak several languages, and a language switch is commonly supported within the application's settings. However, such switching is often incomplete in that it does not perform all changes needed to become relevant to the other language. The *content* of such software is not multilingual, since it is considered to be an input by authors, and authors need to deliver different versions for different languages. To adapt a piece of content, authors are tasked to specifically translate texts and mathematical notations for each language. Typically, this adaptation is done in part with other adjustments that follow the different mathematical traditions. The adaptation of notation often goes as far as changing the letter of variables. For example, the letter *G* is often used in German to denote the summit of a curve (*G* stands for *Gipfel* which means summit), however an English text may use the letter *S* (for *summit*) so as to be as easy as possible to remember in such a sentence as *Let S be the summit of the mountain*.

Dynamic geometry systems are not as flexible as supporting fully the notational differences between languages. One such example is shown in Figure 3, which shows the GeoGebra display of the coordinates of points in the construction. The default language used is English, hence it uses the English notation for numbers and coordinates (comma between coordinates, period as decimal separator) even though the environment is in German (where a vertical bar should be used to separate the coordinates and a comma should be the decimal separator).

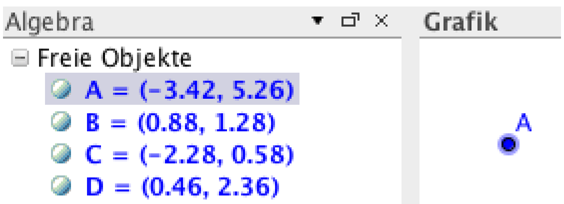

Figure 1: Display of coordinates in GeoGebra in German (but with English notations)

3.3 Multilingualism in Computation Tools

Computers are often seen as tools to perform computations and are particularly useful in performing financial or engineering complex calculations. Computational tools are also commonly used in learning. Three important classes of computational tools used in school mathematics learning are calculators, algebra systems and spreadsheets. The impact of multilingualism on these three is varied.

Calculators are widely used in schools and are mandatory parts of the curriculum in many industrialized countries. Calculators range from basic four-operations calculators to elaborate algebra systems. The calculators' market, until recently, has ignored the language differences with, for example, the only option to press the "." to enter the decimal separator. However a recent trend has appeared in school-oriented calculators where some of the functions are expressed in a local language, including a variation in what symbol to use for decimal separation, depending on the language zone you live in.

Computer algebra systems (CAS) have a similar calculation role as calculators, but with a much richer set of functions and interface. Traditional CASs include the commercial applications such as Mathematica or Maple, and multiple open-source projects such as Sage, Yacas or Macsyma. The set of functions of these systems is so broad that a complete translation almost never exists. Teachers find it normal, to our knowledge, to explain the English names and notations to students. Attempts at delivering multilingual interfaces to computer algebra systems are, however, emerging. Saludes and Xambó (2012) proposed such an approach for Sage. The CATO system proposes a unified user-interface in German for multiple computer algebra systems.[6] We anticipate such moves will continue to grow.

In contrast, interestingly, spreadsheets are generally available only in local languages. They include MicroSoft Excel, OpenOffice Calc and Apple Numbers. They allow the student to arrange data in a table-like fashion and input formula in cells that compute their values from other cells. The formula language does use words of the local language (for example the "SUM" function (in English), taking a range of cell-references, is written as "SOMME" in French or "SUMME" in German). One could speculate that the choice of translating all formulæ matches the expectations that the mathematical knowledge of regular business users is mostly taught in a local language at school. However, probably due to the incomplete capacities of mathematical input on regular keyboards, the language used in such software is far from the normal mathematical notation of everyday school mathematics. It seems that little is known about their language binding which can be a challenge for users who, for example, may search for the appropriate function representing the *average* function, but do not know its name or search by a synonym such as *mean*. To our knowledge, teaching the language of a spreadsheet is often accepted as a duty of mathematics teachers.

3.4 Learning Resources Repositories

Many tools are document based in that they can open documents that authors have created; all the categories above are such. This paradigm, as well as the paradigm of describing a learning scenario by describing the usage of the learning tools, has given rise to the learning resource repositories. These repositories are collections of *educational resources*, which support the teaching and learning by providing access to lesson plans, documents, software, and packages. It is expected that these learning resources can easily be opened and distributed to learners for them to use. A particular family of educational resources are the open educational resources (OERs), which are exchanged together with a license that allows a free redistribution and, often, redistribution in modified form. This family of resources is important, as it allows regular teachers to become part of an economy of exchange of resources, each being adapted to the different needs of the teaching situations.

6 More about the CATO System can be read from http://computeralgebra.biz/.

Together with the software to open and manipulate the resources, repositories support teachers in the differentiated assignment of tasks by endowing them with a broad diversity of resources. Based on various search criteria, they may select learning resources in alternate languages, which they may re-use directly, reuse after a modification, or use as inspiration. This also supports such usage scenarios as the parent trying to work with his or her child in a domain being currently studied at school.

Learning object repositories can also support teachers who are trying to find resources that help their teaching in languages other than the the language of teaching. Indeed, some learning resources are almost instantaneously translatable to another language, even sometimes without understanding the original language. For example the learning resource in Figure 4 from the i2geo.net repository can be used to demonstrate corresponding angles in multiple languages with, at most, the renaming of points (e.g. the use of different characters) or the change of colours.

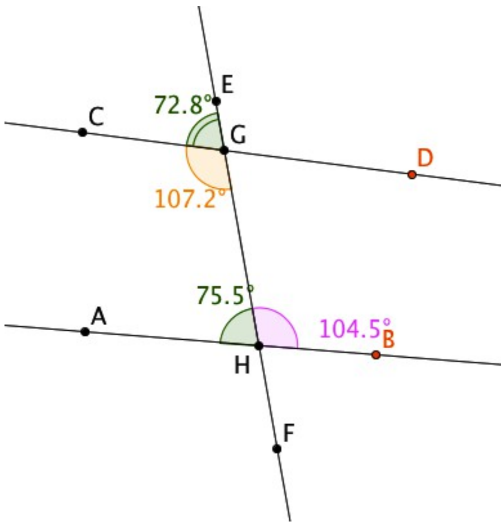

Figure 4: A demonstration of the corresponding angles' concept for the teachers.

National cultures are often not completely defined simply by a common language. Hence adaptations are often needed even when teachers and students might be using the same language. For example, the first author has observed that secondary school teachers in the German state Sachsen-Anhalt require their students to write the names of the vertices of a triangle ordered counter-clockwise, while teachers in the state of Bavaria write them clockwise. The teachers report that they warn their pupils when using resources of the other state since this convention has implications in various reasoning descriptions.

The freedom to perform adaptations is an important technological consequence of using educational resources. It even allows curious teachers to observe the methods of teaching of other cultures and potentially adapt their resources. Moreover, obtaining and then adapting resources is important for teachers who are teaching in regions where resources in their home languages are rare. Of all editorial licensing models we have met, only open licenses seem to allow such a work.

A huge challenge remains in the ability of teachers, learners, or parents who are seeking extra resources for learning mathematics, to formulate appropriate language queries in such a manner that the concepts they are interested in are identified and matched to appropriate learning resources within the resource repositories, or more generally on the web. To this end, the approach of *controlled vocabularies* has been generally used for resource repositories: an editorial team creates a structure of the concepts or domains one expects in the learning resources. This structure enables users to choose from the concepts in this list when searching or contributing, and thus this avoids such ambiguity of using the word *transformation* instead of using *congruence* and allows cross-language queries. Many learning repositories, which display the vocabulary as a hierarchy in each language, use this approach; an example in higher-education mathematics is the merlot.org repository. In the i2geo.net repository, however, one rather searches by entering fragments of the concept name and choosing the appropriate concept as in Figure 5.

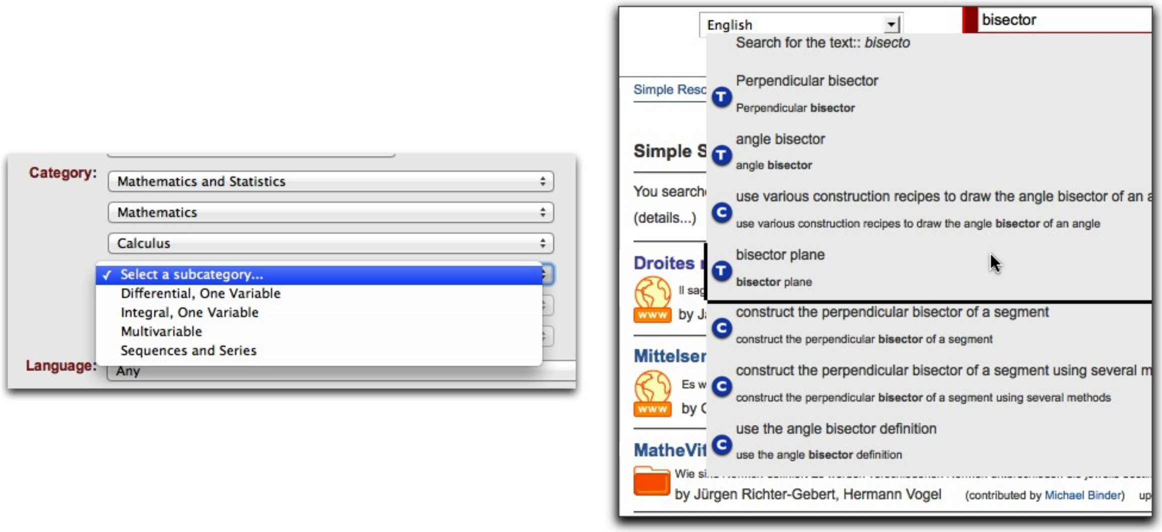

Figure 5: choosing a field in the merlot.org repository or a concept on the i2geo portal.

Both approaches have their strengths and drawbacks. The hierarchy view is often criticized as being to too shallow or too large to read (the example above, for example, requires to know that *sequences and series* is part of *calculus*). On the other hand, with the search approach of i2geo, the users run the risk of not knowing the appropriate words and “miss the right topic”. Nonetheless, both of these approaches succeed in making available search results from multiple languages (since the annotation vocabulary is multi-lingual). In particular, with i2geo.net a few traces teachers that dare to cross the language barriers start to be found.[7]

3.5 Communication Tools

Empowered by worldwide networks, computers, and more recently mobile phones, are being used more and more as communication devices. Written communication in the form of emails for computers and text messages for mobiles are the most widespread. While such devices generally can use the English language, they can often be used to enter other languages as well. The glyph sets and input methods are often available for current languages even though it may be a challenge to acquire, for example, a French speaking operating system in Spain. Most input methods in these tools do not include mathematical

7 An example evaluation from a French teacher on a learning resource in Spanish is at http://i2geo.net/xwiki/bin/view/QR/Coll_msadaall__TransformacionesDeFunciones_3

formulæ; their input often uses quite separate components (e.g. WebEQ applet, Wiris Input Editor, or ASCIIMath widget[8]).

Even without inbuilt tools to facilitate detailed mathematics, these communication tools can be used for many aspects of the mathematical work. For example, questions for assignments can be asked by email or mobile phone short messages. Similarly, web-forum-boards can be used to host discussions about mathematical problems. The multilingual nature of such communication may appear in several forms. However the constraints of entering the questions in a written form, using the limited text available, often requires students and teachers to reformulate the question. These constraints inevitably lead to the invention of a new language. This language adaptation is particularly well known in the usage of basic mobile phones where the input takes considerably shortened forms (e.g. using the word *ur* instead of *your*, which can be mistaken for *you are*, unless the context is carefully considered, as in other language meaning making). Nonetheless, projects such as the one carried out by Waitayangkoon (cited in Kozma, 2005) and the blossoming of school-helpers' industry show that this can provide an important form of support. This space for more focused discussion allows individuals to use a language closer to their own and their peer's informal created language. We contend that this communication can also assume the important role of bringing the school world closer to the homes of students where, often, a different language than the language of teaching is spoken. Indeed, in industrialized countries, the usage of the mobile phone has brought internet-based communication in the daily life of most of the current teenager generation.[9]

Even though the forms of learning that employ mobile phones are still an object of active research, for example in the mLearn series of conferences, they are of importance for the widening of usage of ICTs for learning since the accessibility of these devices is likely to be orders of magnitudes higher than the accessibility of ICTs (Vosloo, 2012).

3.6 Reference Tools

An important and widespread use of the ICTs is the usage of online reference tools in the form of encyclopaedia or dictionaries. Compared to the reference works available in book form, the online reference tools make it possible for both students and teachers to obtain knowledge virtually anytime and anywhere. Although the availability of such resources is sometimes criticized, as it seems to remove an invitation to learn, the online reference tools have made their way into most branches of school teaching. Teachers commonly invite students, if possible, to obtain documentation about a topic. The Wikipedia encyclopaedia (wikipedia.org) is the foremost example of this knowledge source for its richness and coverage. However, as most community-based contribution resources (it is open for anyone to edit), Wikipedia frequently suffers from inhomogeneity and a shallow editorial control, thus it is not rare to find contradicting articles, or articles which only represent the knowledge or practice of a small population. Other dictionaries valuable for school mathematics include the Maths Thesaurus thesaurus.maths.org, the Online Encyclopedia of Integer Sequences (oeis.org), and the MathWorld dictionary (mathworld.com).

8 For the Wiris Input Editor, see http://wiris.com/ , for ASCIImath, see http://www1.chapman.edu/~jipsen/mathml/asciimath.html .

9 At time of writing, two reports are worth mentioning to show the important penetration of the mobile phones in the hands of the young generation: the JIM study in Germany (http://www.mpfs.de/?id=613) which indicates 72% of the 12-19 years old have a smartphone, a number which doubled in 3 years. In the USA, the Mobile Mindset Study (https://www.lookout.com/resources/reports/mobile-mindset) provides similar numbers.

Among these reference tools, only Maths Thesaurus and Wikipedia are multilingual, and this multilingualism is supported by links between the translations. These links are important bridges as they allow acquisition or at least access to another language. Thus when the content given in one language proves to be insufficient, a user first accessing the encyclopaedia in her own language may well then read, for example, the English version, whose coverage could be far more extensive, or indeed may switch to the version in her own language from English for a more thorough and contextual understanding.

Finally, an online reference tool particularly aimed at teachers (and possibly learners) aiming to understand the language differences in the mathematical practices is the Notation Census (Libbrecht, 2010b).[10] This reference tool is a collection of observations from textbooks in multiple languages aimed at recollecting mathematical notations around the world. Persons wishing to know how mathematical concepts are written in different cultures can make use of this to discover the differences. This tool is particularly relevant to combat in a small way the general perception in many cultures that mathematical knowledge is universal and *culture free*. An example of the content of the Notation Census is a comparison of the multiple use of brackets, including the style of bracket, for the half-open interval in the English, French and German, and Dutch languages (Figure 5).

Figure 5: Collected extracts from book-sources in the notation census (redrawn here for the sake of readability) from the census' entry about half-open-interval (http://wiki.math-bridge.org/display/ntns/interval_co).

These notations are not strictly equivalent. This is because they are scans extracted from textbooks. The textbooks are employed as witnesses of the mathematical notations in traditional practice, but like most extractions from original sources, the process of extraction means some diminution of meaning. We anticipate that teachers welcoming students from multiple origins in their classrooms may make use of the Notations Census to help them better understand and probably better explain mathematical concepts. The teachers' explanations may well describe how the symbolic expressions being discussed are used to express, in the students' original languages, the concept in this instance of 'the half-open interval'. Moreover, we anticipate that learning scenarios may be performed by learners who master well a mathematical topic so as to see the concept decoupled from the graphical notations, first visualizing the different practices (and maybe their history), discussing their individual advantages, then inventing their own.

4. Outlook

This chapter has opened a broad set of potential avenues for future research. Within the maturing practices of the ICTs usages for mathematics learning, the multilingual mathematics learners and teachers have a special set of tools that are growing to help them: from localized computational engines to cross-lingual references, from language specific interactive tools to the culture discovery processes empowered by the world-wide-web.

There is little in the mathematics education literature as yet to indicate the effectiveness of these tools for the learners in a multilingual environment. We have presented evidence that we have found available, but clearly more research is needed to answer questions such as the following:

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10 The notation census is available at http://wiki.math-bridge.org/display/ntns/.

- What is needed so that the ICTs used by teachers on a day-to-day basis can provide the language-specific support to individual learners in the modern classroom? In multilingual classrooms, can ICTs tools support all languages?
- Should learners speak multiple mathematical languages so that they are ready to travel or should the tools be localized to their languages?
- At which age and how is it safe to require a student to use, say, an English input syntax to solve and equation or manipulate computation tools? Diverging answers to this question are obtained depending on the expert we have asked it to: Generally, academic mathematicians consider it an important capacity to practice several languages whereas practicing teachers very quickly answer that it is not feasible to teach their students another language.
- What makes a mathematical learning tool easy to use in a given culture? Is it a requirement that the learning tool uses a lexical and notational vocabulary that is consistent with the course? The encoding specificity principle described by Clark and Meyer (2002) seems to indicate so. What is the *cost* of not doing so?

To conclude, we provide an example of a difficult localization issue for which no general theory has been devised. Within the LeActiveMath EU project, an introduction to the concepts of derivative and difference quotient has been written using modelling tasks based on a hike through the mountains where the altitude function was considered. The chapter, including interactive exercises, was written in German and English by a German teacher, with a revision by a team of Scottish teachers. The Scottish teachers judged the content as appropriate and used the material with many of their students. However, what would be needed in order to make such a collection of interactive resources applicable to learners in the USA? Clearly, a conversion of units would be useful since altitude is measured in feet in the USA. But the needed changes are considerably broader than this. A veteran teacher in the state of Indiana in USA indicated that the whole introductory section, including most of the interactive exercises would need to be replaced by something else since mountains are, to students in his state, a concept far away from their vision of the real world.

Is the message from this example that content always needs to be proofed and reviewed for each new culture? This is probably too strong. Is the conclusion that learning software should be disconnected from learning content so as to ease localization? This is more and more impossible in the world-wide-web where content is bound to the interactive tools so that the best consistency of expression is achieved and so that the most interactive learning experience is offered. We also know that learning is much more powerful if conceptualised within a student's context. Hence, if the software is not bound to any content who provides the context? Is it possible really to have learning software devoid of content? Clearly there is still much work needed.

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