Review: Enlightening Symbols, by Joseph Mazur

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Abstract. This book appeared in 2014 at the Princeton University Press.
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Enlightening symbols is a captivating book on the evolution of the mathematical notations. Compared to classical works on the topics, such as F. Cajori's History of mathematical notation, this book is much more of an easy read: it focusses on just a few mathematical notations (the numeric systems, the basic operations, and simple equations) but offers a thorough treatment around themt: the historical context in which the mathematical works were written and read, as well as some graphical extracts of the ancient works. This way, one has an idea how the symbols impacted the way of thinking at that time.

Imagine you would be using words alone to describe and solve polynomial equations in one variable... Who is able to do such a thing today without resorting to writing? That is how the Pythagoreans wrote and communicated similarly as Euclid. For example, the book provides this example: Given a sum of three quantities and also the sums of every pair containing one of those specified quantitys, then that specified quantity is equal to the difference between the sums of those pairs and the total sum of the original three quantities. (first paragraph of chapter 10). The author compares how almost any college student would be able to solve this using x, y, and z (and a, b, c the indicated sums) and how such a recipe, called flower of Thymaridas, was made available at the times of the Pythagoreans so as to solve it.

The book J. Mazur starts with a fairly long exposition on the various number systems: babylonian, greek, roman, hebrew, aztec, chinese, indian... For a few of them, operations are shown with them, and this is where one really meets these numbers, being able to speak about their "capabilities" (e.g. how they can be manipulated to perform long additions, either graphically or on abacus). For a while, the hazardous ways through which the indo-arabic numeral system came through Europe is explored, L. Fibonacci and A. Al Kwaritzmi being among the major carriers. After the numbers, the symbols of early algebra are explored; from word-based algebra, until our x and y, or a and b, quite an amount of imaginative ways have been met which are presented, with the eyes of Diophantus, R. Bombelli, C. Rudolff, or F. Viète. The link to geometry is omnipresent, and indeed this is how a square or a product was considered, but the expressivity of algebra is being developed, until the regularity of combining products of powers of the unknown (adding the exponents), until the resolution of equations becomes easiest, until... the fundamental theorem of algebra and the introduction of the square root of -1. G. Leibnitz and I. Newton conclude the panaché of notations, all leading to the notations currently in use in much of the western world. Then follows a part where less mathematical assertions are made, and more perception and psychology is explored: J. Mazur describes how the perception of symmetry and other patterns influences our perception of formulæ, he describes an experiment he lead, and several other psychologists or neuroscientists, concluding a potential evidence of a sense of reading mathematical formulæ that may be transmitted through evolution.

Unfortunately, the book does not mention at all the differences of notations across the cultures and languages (such as the 10 ways of doing the long division, or the usage of j for the root of -1 in electrical engineering). However, this is a rather natural consequence of the book: notations have evolved differently depending on the usages. Neither does the book offer sufficient material for proposing problems to students. However, most of the references are well documented, with URLs when possible. This should allow a teacher to go and understand the ancient works so as to propose introductory and exercise materials. Certainly such a continuation of the book would be very interesting to share in a wiki space such as the census of mathematical notations (http://wiki.math-bridge.org/display/ntns/).

A tiny note to the readers who, similarly to me, like to read the electronic versions of the book: reading on a small mobile device can be made well as the eBook version is available (e.g. from Google Play Books as ePub and PDF and many others). However, some characters may be missing on the mobile version (such as the inverse psi to indicate a minus in Diophantus times); moreover, the ePub version suffers from an amount of missing characters in the formulæ and words, with which normal mathematicians can easily cope but need some vigilance.

Overall, i recommend the reading of this book to anyone thinking about the mathematical notation they use and could use. It may even be a reading for late college students. For those who think that we should stop discussing or varying mathematical notations, there is certainly lots of food for thoughts to illustrate how deeply the notation influences our conceptualization of mathematics as "Perheaps, but routine and familiarity are the tailwind of conceptions" (last paragraph of chapter 10).

Let me conclude this review with one of the many precious sentences of this book, one by its author when describing the introduction of the complex numbers made of a real and an imaginary part: "Unfortunately, because they are the names of classes of numbers that are neither imaginary nor complex." (paragraph 32 of chapter 20).